

Quantum censorship in two dimensions

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It is pointed out that increasingly attractive interactions, represented by partially concave local potential in the Lagrangian, may lead to the degeneracy of the blocked, renormalized action at the gliding cutoff scale by tree-level renormalization. A quantum counterpart of this mechanism is presented in the two-dimensional sine-Gordon model. The presence of Quantum Censorship is conjectured which makes the loop contributions pile up during the renormalization and thereby realize an approximate semiclassical effect.

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Introduction The renormalized trajectory of a quantum system maps out the scale dependence of the effective parameters of the system. It is one of the basic tenets of the renormalization group procedure that the critical behaviors, the singular dependence of the IR observables on the UV parameters, builds up by scanning through infinitely wide scale regions rather by a singularity observed at a finite scale. Correspondingly, the renormalized trajectory should at least be continuous in the scale parameter, an expectation which has already been justified for local quantum field theories [1]. But the continuity of the renormalized trajectory in the cut-off does not exclude other singularities.

Let us consider an Euclidean theory characterized by the action $S_k[\phi]$, k being the sharp UV cutoff, and write the field variable as $\phi + \phi'$ where the supports of ϕ and ϕ' in Fourier space are $|p| < k - \Delta k$ and $k - \Delta k < |p| < k$, respectively. An infinitesimal blocking step corresponds to integrating out the modes close to the cutoff, giving the functional integration

$$e^{-\frac{1}{\hbar}S_{k-\Delta k}[\phi]} = \int D[\phi'] e^{-\frac{1}{\hbar}S_k[\phi+\phi']}, \quad (1)$$

which may possess a saddle point. The derivatives of the trajectory with respect to the cutoff are obviously singular at the scale where this saddle point appears or disappears. Such a tree-level renormalization has been found in the spinodal instability (SI) regions and was responsible for the degeneracy of the blocked action at the cutoff scale for certain homogeneous background field ϕ in the ϕ^4 and the sine-Gordon (SG) model [2, 3], a dynamical generalization of the Maxwell-cut.

The blocking Eq. (1) yields a functional finite difference equation whose solution lies well beyond our analytical capabilities. It is usually handled by imposing rather simple restrictions, either by ignoring altogether the loop contributions to the blocking or restricting the evolution of the action into few coupling constants. In the context of the SG model one retains some Fourier coefficients of $V_k(\phi)$ in the local potential approximation [3],

$$S_k[\phi] = \int_x \left[\frac{1}{2} (\partial \phi_x)^2 + V_k(\phi_x) \right]. \quad (2)$$

The saddle points considered were plane waves and the degeneracy of the action for modes at the cutoff was found by recovering the potential $V_k(\phi) = -k^2 \phi^2/2$ in the SI region, in a certain interval for ϕ . The inhomogeneous saddle points generate non-perturbative soft modes, the zero modes corresponding to the breakdown of the external, space-time symmetries. Beyond these intervals for the field the theory appeared to be stable, without unexpected soft modes. At the end of the intervals, at the border of the stable and unstable regions the loop corrections make the potential non-analytical [2] and the truncation of the potential, behind any expansion scheme, becomes highly suspicious.

Beyond the problem of justifying the omission of the loop correction or the possible non-analytical structures in the potential there is an even more fundamental issue here. The finite difference equation Eq. (1) contains infinitely many higher loop contributions which are suppressed by the small parameter $\epsilon_k = \hbar \Delta k/k |\ln \lambda_{\min}|$, λ_{\min} being the smallest eigenvalue of $\delta^2 S[\phi]/\delta \phi \delta \phi$. The availability of the loop expansion is assumed in deriving the evolution equation which in turn resums the expansion in the differential equation limit, $\Delta k/k \rightarrow 0$. There are two ways ϵ_k can become large, either for $k \rightarrow 0$ or for $\lambda_{\min} \rightarrow 0$. The first possibility, a singular thermodynamical limit, is discarded for the usual, local models. The second alternative, the case of degenerate action is more realistic. Once the blocked action becomes exactly degenerate at the cutoff scale where it is supposed to describe best the dynamics then the integral in Eq. (1) obeys no expansion anymore and we have no analytical tool left to tackle the problem. Therefore, certain singularities of the renormalized trajectory, such as the degeneracy of the action may have serious consequences.

This leads to the obvious question, whether higher order loop corrections influencing the global properties of the potential or further higher order terms of the action in the space-time derivatives can prevent the degeneracy at some finite value of the gliding cutoff scale. Guided by an analogous problem in General Relativity such a mechanism might be called the Quantum Censorship (QC) in Quantum Field Theory [4]. This issue is considered here

in the framework of the two-dimensional SG model, a choice to be motivated later, by solving the evolution equation Eq.(1) numerically within the local potential approximation for a general potential, without any restriction imposed on it.

SG model The theory considered in this work is defined by the action of Eq.(2) in two-dimensional Euclidean space-time where the bare potential at the initial cut-off, $k_{\text{init}} = \Lambda = 1$, is $V_B(\Phi) = k^2 \tilde{u}_B \cos(\sqrt{8\pi}\beta_r \Phi)$. It exhibits a Z_2 symmetry $\phi(x) \rightarrow -\phi(x)$ and periodicity in the internal space, $\phi(x) \rightarrow \phi(x) + 2\pi/\beta$ ($\beta = \sqrt{8\pi}\beta_r$). The evolution of the local potential is governed by the Wegner-Houghton equation [5] in $d = 2$

$$(2 + k\partial_k)\tilde{V}_k(\phi) = -\frac{1}{4\pi} \ln \left(1 + \tilde{V}_k''(\phi) \right), \quad (3)$$

in terms of the dimensionless potential $\tilde{V}_k = k^{-2}V_k$, in the absence of the saddle point in Eq.(1). Note that the argument of the logarithm function is the restoring force acting on the quantum fluctuations at the cutoff and should be positive to justify the loop expansion. When the argument becomes negative then this equation is not valid anymore and the saddle point contributions to the integral of Eq.(1) have to be taken in account.

The model is known to exhibit two phases [3, 6, 7, 8], separated by the Coleman point, $\beta_r = 1$. The simplest indication is the change of the sign of the one-loop beta-function for \tilde{u} at this point, $\tilde{u}_k \approx \tilde{u}_B(k^2/\Lambda^2)^{\beta_r-1}$. The phase $\beta_r > 1$ or $\beta_r < 1$ preserves or breaks the internal symmetries, respectively. The phase with broken symmetry is equivalent with the neutral sector of the massive Thirring model [6] and the neutral Coulomb-gas [9]. The lattice regulated SG model can be mapped to the planar XY model [10] providing a non-perturbative renormalization group (RG) flow.

The symmetry broken phase contains further special points. Higher, n -th order perturbative contributions generate the potential $u_n \cos(\sqrt{8\pi}\beta_r n\Phi)$ and the corresponding coupling strength is renormalizable or non-renormalizable for $\beta_r < \beta_r^{(n)} = 1/n$ or $\beta_r > \beta_r^{(n)}$, respectively. Therefore, the theory with $\beta_r^{(n+1)} < \beta_r \leq \beta_r^{(n)}$ has n perturbatively renormalizable parameters apart of β . The UV scaling regime was found to be very limited due to intermediate scaling laws, appearing in between the UV and the approximate SI scaling regimes where all coupling constants grow with decreasing cutoff [8]. The point $\beta_r^{(4)}$ deserves special attention. The duality established in the Villain model [11], $(\beta, u, z) \rightarrow (2\pi/\beta, 2z, u/2)$ where z denotes the vortex fugacity, maps the XY model without external field, $u = 0$, into the continuum SG model [10]. A special feature of the continuum SG model is its non-periodic kinetic energy which suppresses the vortices in the XY model context, configurations with point singularity. Thus the $z = 0$ plane corresponds to the continuum SG model, studied in this work. The dual of the

Coleman-point $\beta_r = 1$ is the Kosterlitz-Thouless critical point, $\beta_r^{(4)}$.

The SG model is similar to non-Abelian gauge theories in what the field variable is compact. The effective potential of a compact variable is flat, the constant being the only function which is periodic and convex in the same time. Therefore, the effective potential, $V_{k=0}(\Phi)$ can not distinguish the phases and one expects similar phenomena in non-Abelian gauge theories, as well. It was found that the potential $\tilde{V}_k(\Phi)$, expressed in units of the cutoff, solves this problem and can be used to identify the phase structure [3].

It is natural to represent the periodic potential of the SG model by a Fourier series. But the Fourier series of the potential $V_k^{SI}(\phi) = -k^2\phi^2/2$ for $-\pi/\beta < \phi < \pi/\beta$ with its periodic extension seen approximately in the IR region by following the evolution of a truncated Fourier series [8], converges badly and all we can ascertain is that an approximate degeneracy occurs in the symmetry broken phase. Note that the Fourier-expansion based numerical solution of different evolution equations [12, 13, 14, 15] suffers the same problem and it is difficult to decide whether the SI occurs or not.

We avoid the limitations of a truncated series by solving the evolution equation for unconstrained potential numerically. The potential is represented in the algorithm by spline polynomials [4]. The evolution of the coefficients of the Chebyshev polynomials are followed in this method and the linear algebra employed becomes singular for degenerate actions. The internal consistency checks of the algorithm, controlling the derivatives and the integration adjust the step size, Δk dynamically and stops the execution of the computer program when the a partial error in the algorithm reaches the precision of the number representation in the computer.

Coleman point The theory with $\beta_r = 1$ separates two phases [6]. The RG flow in the $\beta_r > 1$ phase which is usually referred to as the non-renormalizable phase gives a simple evolution due to the smallness of \tilde{u} . What is more interesting for us is that the potential barrier between two neighboring minima is thinner for large β_r and the fluctuations can "fill up" the minima easier. The result is QC, the stable, loop-generated and gradual approach of the potential $V_k^{SI}(\phi)$ as the cutoff is lowered with the establishment of exactly degenerate action for $k \rightarrow 0$ only. This scenario was actually established in this phase by following the evolution of the local potential, represented by a truncated Fourier expansion [8, 16]. The small β_r , or renormalizable phase shows an interesting, more involved structure because the barrier between the minima of the potential is wider and QC is more difficult to realize. The approximation [8, 16] indeed leads here to degenerate action and to tree-level renormalization and QC is prevented to act. The result is a super-universal, strictly bare parameter independent local po-

tential. The numerical solution of the evolution equation

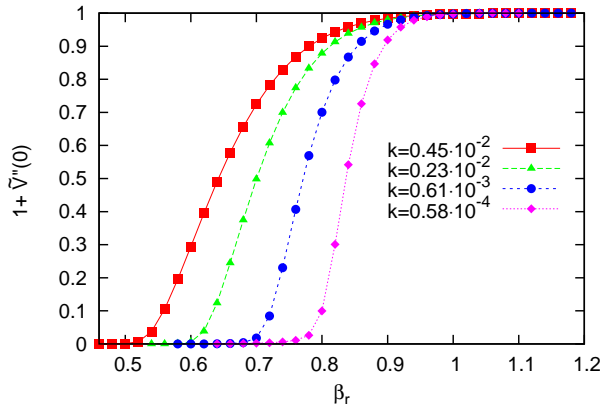


FIG. 1: The second derivative of the action is plotted as the function of the relative parameter β_r , for various values of the scale k . It goes to approximately zero when $\beta_r < 1$ and to 1 when $\beta_r > 1$.

with unconstrained potential confirms the flattening of the dimensionless potential for $\beta_r > 1$, as one can see in Fig. 1 where $1 + \tilde{V}''(\phi = 0)$ is plotted for different values of the cutoff k . What we see in the symmetry broken phase, $\beta_r < 1$, is that the action is nearly degenerate, $\tilde{V}''(0) \approx 0$, the argument of the logarithm function in Eq. (3) is nearly vanishing and the evolution is very close to be singular.

Descent in β What is the fate of QC in the small β_r phase? As mentioned above, neither analytical nor numerical methods are available to our knowledge to answer this question. No analytical method is known to regulate and to handle functional integrals with constant integrands. Even if we grant the evolution equation Eq. (3), no numerical algorithm, realized by finite computing power can distinguish exact degeneracy from small but finite variation. What is left is to collect circumstantial evidences to support our conjecture, spelled out below.

We start on the analytical side, by noting that the almost degeneracy of the action at the cutoff as shown in Fig. 1 generates large amplitude, non-perturbative modes with small but finite wave numbers, the hallmark of spontaneous symmetry breaking. It is the fundamental group symmetry which might break at this point, the invariance of the theory with respect to the shift $\phi(x) \rightarrow \phi(x) + 2\pi/\beta$. The breakdown of this symmetry is realized by restricting the functional integral over field configurations with a given number of topological charge. In fact, the derivation of the correct Schwinger-Dyson hierarchical equations, obtained formally by performing infinitesimal variation of the field variables, requires to integrate in the path integral over a field-configuration space which is closed under smooth deformations. The smallest domain satisfying this condition consists of a single homotopy class, describing the propagation of a fixed

number of kinks. The spontaneous breakdown of the fundamental group symmetry is realized by the dynamical restriction of the integration domain of the path integral for infinitely large systems. The SG model is asymptotically free in the UV limit in the symmetry broken phase and its semiclassical solution reveals stable kinks. The important message of this line of thought from our point of view is that the dynamical stability of kinks lends stability for certain inhomogeneous configurations in the blocking, Eq. (1) and thereby may prevent QC to be realized. The numerical results confirm this tendency. The

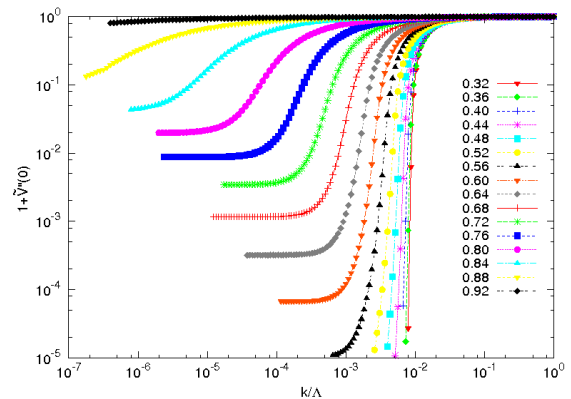


FIG. 2: The saturation of the curvature at small k for various values of β_r . The value of β_r are shown for each curve.

scale dependence of the degeneracy $1 + \tilde{V}''(0)$, depicted in Fig. 2, indicates that the loop corrections renormalize the action to a non-degenerate, scale invariant form below a crossover scale which moves in the IR direction as we penetrate into the symmetry broken phase. This result, not foreseen in the previous RG calculations, requires an unconstrained treatment of the local potential because infinitely many Fourier modes mix during the evolution and render any truncated scheme invalid. The horizontal segments in Fig. 2 signal that QC operates for $\beta_r^{(2)} < \beta_r < \beta_r^{(1)}$ but with a strength which decreases with β_r . The decrease of QC forces $1 + \tilde{V}''(0)$ to drop earlier during the evolution which increases the scale windows of the almost degenerate action. For $\beta_r < \beta_r^{(2)}$ either the trajectory is stabilized at a degeneracy level which is not detectable within the accuracy of the double precision number representation or the steep drop of the degeneracy continues until we hit true degeneracy and eventually generate saddle points. The degree of degeneracy of the action can be further explored by means of the sensitivity matrix whose elements are the derivatives of the renormalized quantity $\tilde{V}_k''(0)$ with respect to the bare parameters, in particular $S_k = \partial \tilde{V}_k''(0) / \partial \beta_r$ in the present case which is shown in Fig. 3. For approximately $\beta_r > \beta_r^{(4)}$ it starts with negative values at the cutoff (not visible in the Figure) but traverses zero and becomes positive at a scale which moves in the UV di-

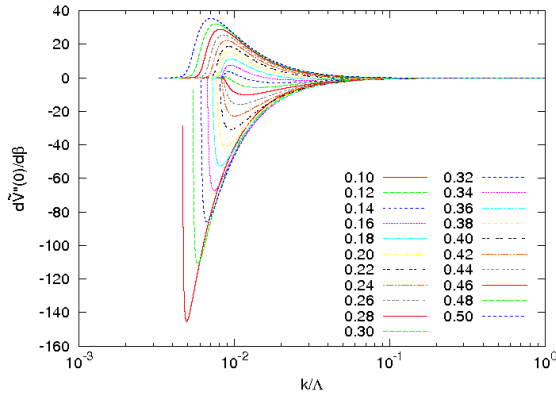


FIG. 3: The sensitivity matrix element up to a sign, S_0 , as the function of the scale for various β_r . The value of β_r are shown for each curve.

rection as β_r is raised. For $\beta^{(3)} < \beta_r < \beta^{(2)}$ the positive peak is higher and the negative one is lower than one. For $\beta^{(4)} < \beta_r < \beta^{(3)}$ the two peaks have comparable heights. Finally, for approximately $\beta_r < \beta^{(4)}$ S_k stays negative and develops a strong peak. Such a dependence is consistent with a strengthening degeneracy as β_r is decreased between $\beta^{(2)}$ and $\beta^{(3)}$. The further decrease of β_r seems to increase the degeneracy even more. The dominant, IR part of the matrix element obeys the approximate scaling law $S_k \approx \pm 1/k^2$ with a high accuracy until the evolution changes abruptly for $\beta_r < \beta^{(4)}$. It is far from clear if this is a precursor of a turn towards degeneracy.

Note that the appearance of the special values $\beta_r^{(n)}$ in the features discussed above is natural, these are the β_r values where the UV critical exponent of a coupling constant changes sign, altering in a profound way the competition between the various Fourier modes in approaching the degenerate action.

Summary It is pointed out in this paper that characteristically classical dynamics, such as the classical collective coordinate generated Maxwell-cut, might be mimicked by quantum fluctuations with a surprising accuracy. Such a smearing of the usual singularities of the tree-level contributions is called QC.

The thumb rule to estimate the strength of QC is to find the strength of fluctuations which may have similar effects than the classical saddle points. Note that the functional integration in question, the blocking Eq. (1), is over an UV subspace of configurations ϕ' only and the IR background field ϕ can stabilize inhomogeneous saddle points even if the full functional integral of the theory possesses no such saddle points. The fluctuations around the saddle points are strong in general if the action has a shallow minimum at the saddle point. The

fluctuations which may wash different saddle points together are strong if different saddle points are close in the field space. The distinguished feature of the SG model is the periodicity of its potential which allows us to control the latter type of fluctuations by the parameter β_r and makes this model a good testing ground for QC.

Circumstantial evidences were presented for the gradual weakening QC in the two-dimensional SG model as the period length of the potential is increased in the field space. But the final word about the fate of QC in the small β_r part of the phase diagram remains a provocative open problem.

We have considered vacuum expectation values in this work from the point of view of QC. Another issue what remains open whether the full classical behavior, decoherence included can be reproduced by QC. We plan to return to this problem in a future publication.

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- [1] R. B. Israel, in *Random Fields* (Esztergom, 1979), J. Fritz, J. L. Lebowitz, D. Szász eds. (North-Holland, Amsterdam, 1981); A. C. D. van Enter, R. Fernández, A. Sokal, *J. Stat. Phys.* **72**, 879 (1993).
 - [2] J. Alexandre, V. Branchina, J. Polonyi, *Phys. Lett. B* **445**, 153 (1999).
 - [3] I. Nándori, J. Polonyi, K. Sailer *Phys. Rev. D* **63**, 045022 (2001).
 - [4] V. Pagon, S. Nagy, J. Polonyi, K. Sailer, e-print: hep-th/0907.0144, submitted to *Phys. Rev. D*
 - [5] F. J. Wegner, A. Houghton, *Phys. Rev. A* **8**, 401 (1973).
 - [6] S. R. Coleman, *Phys. Rev. D* **11**, 2088 (1975);
 - [7] S. Mandelstam, *Phys. Rev. D* **11**, 3026 (1975); B. Schroer, T. Truong, *Phys. Rev. D* **15**, 1684 (1977); D. J. Amit, Y. Y. Goldschmidt and G. Grinstein, *J. Phys A* **13**, 585 (1980); A. I. B. Zamolodchikov, *Int. J. Mod. Phys. A* **10**, 1125 (1995); J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, (Clarendon Press, Oxford, 1996).
 - [8] S. Nagy, I. Nándori, J. Polonyi, K. Sailer, *Phys. Lett. B* **647**, 152 (2007); *Phys. Rev. Lett.* **102**, 241603 (2009).
 - [9] S. Samuel, *Phys. Rev. D* **18**, 1916 (1978).
 - [10] K. Huang, J. Polonyi, *Int. J. of Mod. Phys. B* **6**, 409 (1991).
 - [11] J. V. José, L. P. Kadanoff, S. Kirkpatrick, D. R. Nelson, *Phys. Rev. B* **16**, 1217 (1977).
 - [12] J. Polchinski, *Nucl. Phys B* **231**, 269 (1984).
 - [13] T. R. Morris, *Int. J. Mod. Phys. A* **9**, 2411 (1994); M. E. Fisher, *Rev. Mod. Phys.* **70**, 653 (1998); C. Bagnuls, C. Bervillier, *Phys. Rept.* **348**, 91, (2001); J. Berges, N. Tetradis, C. Wetterich, *Phys. Rept.* **363**, 223 (2002).
 - [14] J. Polonyi, *Central Eur. J. Phys.* **1**, 1 (2003).
 - [15] J. Pawłowski, *Annals Phys.* **322**, 2831 (2007).
 - [16] S. Nagy, I. Nándori, J. Polonyi, K. Sailer, *Phys. Rev. D* **77**, 025026 (2008).